1 AP & GP

1.1 Series

Let $u_1, u_2, ...u_n$ be a sequence

then $S_n = u_1 + u_2 + u_3 + ... + u_n$

Result $u_1 = S_1$, $u_n = S_n - S_{n-1}$

In summation form: $S_n = \sum_{i=1}^{n} u_i$

1.2 Arithmetic Series

Arithmetic Progression: $a, a+d, a+2d, ...$

Common Difference: $d = u_n - u_{n-1}$

Nth Term: $u_n = a + (n-1)d$

Sum of Sequence: $\frac{n}{2}(u_1 + u_n) = \frac{n}{2}[2a + (n-1)d]$

1.3 Geometric Series

Geometric Progression: $a, ar, ar^2, ar^3, ...$

Common Ratio: $r = \frac{u_2}{u_1} = \frac{u_3}{u_2} = ... = \frac{u_n}{u_n - 1}$

Nth Term: $u_n = ar^{n-1}$

Sum: $S_n = \frac{a}{1-r}(1 - r^n)$, $r \neq 1$ when $r = 1, S_n = na$

Sum to infinite: for $-1 < r < 1$, $S_\infty = \frac{a}{1-r}$

1.4 Binomial Theorem

Coeff: $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

Theorem: $(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + ... + \binom{n}{n}a^0 b^n$

Generalized Coeff: $\binom{n}{r} = \frac{n(n-1)(n-2)...(n-r+1)}{r!}$

E.g. $\binom{\frac{1}{2}}{3} = \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2}))}{3!}$

Generalized Theorem: $(1+a)^n = 1 + na + \frac{n(n-1)}{2!}a^2 + ...$
when $n < 0\, and -1 < a < 1$

Telescoping Series: $\sum_{r=m}^{n}(a_r - a_{r\pm1})$

2 Differentiation

Function	Differential
$(f(x))^n$	$nf'(x)(f(x))^{n-1}$
$\cos(x)$	$-\sin(x)$
$\sin(x)$	$\cos(x)$
$\tan(x)$	$\sec^2(x)$
$\sec(x)$	$\sec(x)\tan(x)$
$\csc(x)$	$-\csc(x)\cot(x)$
$\cot(x)$	$-\csc^2(x)$
$e^{f(x)}$	$f'(x)e^{f(x)}$
$\ln(f(x))$	$\frac{f'(x)}{f(x)}$
$\sin^{-1}(f(x))$	$\frac{f'(x)}{\sqrt{1-f(x)^2}}$
$\cos^{-1}(f(x))$	$-\frac{f'(x)}{\sqrt{1-f(x)^2}}$
$\tan^{-1}(f(x))$	$\frac{f'(x)}{1+f(x)^2}$

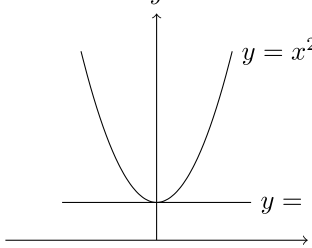
Product Rule: $\frac{d}{dx}(ab) = \frac{da}{dx}(b) + \frac{db}{dx}(a)$
Quotient Rule: $\frac{d}{dx}(\frac{a}{b}) = \frac{\frac{da}{dx}(b) - \frac{db}{dx}(a)}{b^2}$
Chain Rule: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Implicit: $\frac{d}{dx}(y^n) = ny^{n-1}\frac{dy}{dx}$
$y = f(x)^{g(x)}$
$\ln(y) = g(x)\ln(f(x))$

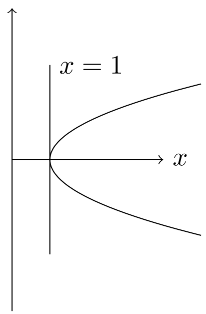
$\frac{d}{dx}(a^x) = a^x ln(a) \times \frac{d}{dx}(x)$

$\frac{d^2y}{dx^2} = \frac{d}{du}(\frac{dy}{dx}) \times \frac{du}{dx}$

Equation of tangent: $y - y_0 = m(x - x_0)$
Equation of normal: $y - y_0 = -\frac{1}{m}(x - x_0)$



Tangent // x-axis, $\frac{dy}{dx} = 0$



Tangent // y-axis, $\frac{dy}{dx} = \pm\infty$

If $f \approx a, f(x) \approx f'(a)[x-a] + f(a)$
If $f'(x) > 0$ it is increasing, else decreasing
If $f''(x) > 0$ it is concave up, else concave down

If $f'(x) = 0\ \&f''(x) < 0$ it is local maximum
If $f'(x) = 0\ \&f''(x) > 0$ it is local minimum
If $f'(x) = 0\ \&f''(x) = 0$ test fails

2.1 Trigonometric Identities

$\sin^2\theta + \cos^2\theta = 1$
$\tan^2\theta + 1 = \sec^2\theta$
$1 + \cot^2\theta = \csc^2\theta$
$\sin 2\theta = 2\sin\theta\cos\theta$
$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$\cos 2\theta = 2\cos^2\theta - 1$
$\cos 2\theta = 1 - 2\sin^2\theta$
$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$
$\sin(\alpha+\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
$\cos(\alpha+\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

3 Integration

3.1 Standard Integrals

1	$\int (ax+b)^n \, dx$	$\frac{(ax+b)^{n+1}}{(n+1)a} + C$
2	$\int \frac{1}{ax+b} \, dx$	$\frac{1}{a} \ln\lvert ax+b\rvert + C$
3	$\int e^{ax+b} \, dx$	$\frac{1}{a} e^{ax+b} + C$
4	$\int \sin(ax+b) \, dx$	$-\frac{1}{a}\cos(ax+b) + C$
5	$\int \cos(ax+b) \, dx$	$\frac{1}{a}\sin(ax+b) + C$
6	$\int \tan(ax+b) \, dx$	$\frac{1}{a}\ln\lvert\sec(ax+b)\rvert + C$
7	$\int \sec(ax+b) \, dx$	$\frac{1}{a}\ln\lvert\sec(ax+b)+\tan(ax+b)\rvert + C$
8	$\int \csc(ax+b) \, dx$	$-\frac{1}{a}\ln\lvert\csc(ax+b)+\cot(ax+b)\rvert + C$
9	$\int \cot(ax+b) \, dx$	$-\frac{1}{a}\ln\lvert\csc(ax+b)\rvert + C$
10	$\int \sec^2(ax+b) \, dx$	$\frac{1}{a}\tan(ax+b) + C$
11	$\int \csc^2(ax+b) \, dx$	$-\frac{1}{a}\cot(ax+b) + C$
12	$\int \sec(ax+b)\cdot\tan(ax+b) \, dx$	$\frac{1}{a}\sec(ax+b) + C$
13	$\int \csc(ax+b)\cdot\cot(ax+b) \, dx$	$-\frac{1}{a}\csc(ax+b) + C$
14	$\int \frac{1}{a^2+(x+b)^2} \, dx$	$\frac{1}{a}\tan^{-1}(\frac{x+b}{a}) + C$
15	$\int \frac{1}{\sqrt{a^2-(x+b)^2}} \, dx$	$\sin^{-1}(\frac{x+b}{a}) + C$
16	$\int \frac{-1}{\sqrt{a^2-(x+b)^2}} \, dx$	$\cos^{-1}(\frac{x+b}{a}) + C$
17	$\int \frac{1}{a^2-(x+b)^2} \, dx$	$\frac{1}{2a}\ln\lvert\frac{x+b+a}{x+b-a}\rvert + C$
18	$\int \frac{1}{(x+b)^2-a^2} \, dx$	$\frac{1}{2a}\ln\lvert\frac{x+b-a}{x+b+a}\rvert + C$
19	$\int \frac{1}{\sqrt{(x+b)^2+a^2}} \, dx$	$\ln\lvert(x+b)+\sqrt{(x+b)^2+a^2}\rvert + C$
20	$\int \frac{1}{\sqrt{(x+b)^2-a^2}} \, dx$	$\ln\lvert(x+b)+\sqrt{(x+b)^2-a^2}\rvert + C$
21	$\int \frac{1}{\sqrt{(x+b)^2-a^2}} \, dx$	$\ln\lvert(x+b)+\sqrt{(x+b)^2-a^2}\rvert + C$
21	$\int a^x \, dx$	$\frac{a^x}{\ln a} + C$

3.2 Integration by Parts

$\int u \, dv = uv - \int v \, du$

Rule for choosing u

Logarithm	$\ln(ax+b)$
Inverse Trigo	$\sin^{-1}(ax+b)$
Algebraic	x, x^{10}
Trigo	$\sin(ax+b)$
Expo	$e^x, 19^x$

3.3 Area between 2 curves

$A = \int_a^b g(x) - f(x)dx,$ when $g(x)$ is above $f(x)$

3.4 Volume of Revolution

$V = \pi \int_a^b (f(x)-a)^2 \, dx$ when a is a line parallel to x or axis

$V = \pi \int_a^b (f(x))^2 \, dx - \pi \int_a^b (g(x))^2 \, dx$ when $f(x)$ is higher than $g(x)$

4 Vectors

$\overrightarrow{OA} = a = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = x_1\mathrm{i} + y_1\mathrm{j} + z_1\mathrm{k}$

$\overrightarrow{OB} = b = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = x_2\mathrm{i} + y_2\mathrm{j} + z_2\mathrm{k}$

Magnitude = $|\overrightarrow{AB}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ = $\begin{pmatrix} x_2-x_1 \\ y_2-y_1 \end{pmatrix}$

Unit Vector : $\hat{v} = \frac{1}{|v|} v$

Dot Product: $a \cdot b = x_1x_2 + y_1y_2 + z_1z_2 = |a||b|\cos\theta$

If $a \perp b$, $a \cdot b = 0$

$\theta = \cos^{-1}\left(\frac{a\cdot b}{|a||b|}\right)$

Cross Product: $a \times b = \begin{pmatrix} y_1z_2 - y_2z_1 \\ -(x_1z_2 - x_2z_2) \\ x_1y_2 - x_2y_1) \end{pmatrix}$

Area of $\triangle ABC = \frac{1}{2}|\overrightarrow{CA} \times \overrightarrow{CB}|$

$|a \times b| = |a||b|\sin\theta$

Line: $r = a + \lambda u \Leftrightarrow r = (x_1\mathrm{i} + y_1\mathrm{j} + z_1\mathrm{k}) + t(a\mathrm{i} + b\mathrm{j} + c\mathrm{k})$ where a is a point and u is a direction vector

If Point $P \perp$ to line $r = a + s\overrightarrow{u}$, $Q = (a + \lambda\overrightarrow{u})$, $\overrightarrow{PQ} \cdot \overrightarrow{u} = 0$

Shortest distance = $|PQ|$

Plane: $(\overrightarrow{r} - \overrightarrow{a}) \cdot n = 0 \Leftrightarrow \overrightarrow{r} \cdot \overrightarrow{n} = \overrightarrow{a} \cdot \overrightarrow{n}$, where a and r are 2 vectors on the plane and n is normal to the plane

Cartesian Eqn of plane: $r \cdot n = d \Leftrightarrow ax + by + cz = d$, where $n = ai + bj + ck$ and $r = xi + yj + zk$

Angle between planes: $\cos\theta = |\frac{n_1 \cdot n_2}{|n_1||n_2|}|$ Angle between line and plane: $\sin\theta = |\frac{u\cdot n}{|u||n|}|$

Intersection of 2 planes: $r = a + \lambda(n_1 \times n_2)$