MA1522

Assignment 2

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1 Question 1

$$\overrightarrow{A} = \begin{pmatrix} 2 & -4 & 4 & -2 \\ 6 & a-12 & 7 & a-6 \\ -1 & 2-b & 8 & -b+1 \end{pmatrix} \xrightarrow[R_3+\frac{1}{2}R_1]{R_2+(-3)R_1} \begin{pmatrix} 2 & -4 & 4 & -2 \\ 0 & a & -5 & a \\ 0 & -b & 10 & -b \end{pmatrix}$$

$$\xrightarrow{R_3+\frac{b}{a}R_2} \begin{pmatrix} 2 & -4 & 4 & -2 \\ 0 & a & -5 & a \\ 0 & 0 & 10-\frac{5b}{a} & 0 \end{pmatrix} = \overrightarrow{U}$$

$$\overrightarrow{I} \xrightarrow{R_3+\frac{-b}{a}R_2} \xrightarrow[R_3+\frac{-1}{2}R_1]{R_2+(3)R_1} \overrightarrow{L}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_3+(-\frac{b}{a})R_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{b}{a} & 1 \end{pmatrix} \xrightarrow[R_3+\frac{-1}{2}R_1]{R_2+(3)R_1} \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -\frac{1}{2} & -\frac{b}{a} & 1 \end{pmatrix} = \overrightarrow{L}$$

If $a = 0$, then in the 1st step of the factorization above, it will cause the array to pivot. Thus, to prevent pivoting, $b = 0$ must be the case also. $a \neq 0$ then $b \neq 0$ also, as $\frac{b}{a}$ will not be defined

2 Question 2

For A to be invertible, $det(A) \neq 0$.

$$A = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & c \\ 1 & 4 & 1 & c^2 \\ 1 & 8 & -1 & c^3 \end{vmatrix}$$

$$\begin{aligned}
det(A) &= (1)\begin{vmatrix} 2 & -1 & c \\ 4 & 1 & c^2 \\ 8 & -1 & c^3 \end{vmatrix} + (-1)\begin{vmatrix} 1 & -1 & c \\ 1 & 1 & c^2 \\ 1 & -1 & c^3 \end{vmatrix} + (1)\begin{vmatrix} 1 & 2 & c \\ 1 & 4 & c^2 \\ 1 & 8 & c^3 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 2 & -1 \\ 1 & 4 & 1 \\ 1 & 8 & -1 \end{vmatrix} \\
&= (1)\left((2)\begin{vmatrix} 1 & c^2 \\ -1 & c^3 \end{vmatrix} + (1)\begin{vmatrix} 4 & c^2 \\ 8 & c^3 \end{vmatrix} + (c)\begin{vmatrix} 4 & 1 \\ 8 & -1 \end{vmatrix}\right) \\
&+ (-1)\left((1)\begin{vmatrix} 1 & c^2 \\ -1 & c^3 \end{vmatrix} + (1)\begin{vmatrix} 1 & c^2 \\ 1 & c^3 \end{vmatrix} + (c)\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}\right) \\
&+ (1)\left((1)\begin{vmatrix} 4 & c^2 \\ 8 & c^3 \end{vmatrix} + (-2)\begin{vmatrix} 1 & c^2 \\ 1 & c^3 \end{vmatrix} + (c)\begin{vmatrix} 1 & 4 \\ 1 & 8 \end{vmatrix}\right) \\
&+ (-1)\left((1)\begin{vmatrix} 4 & 1 \\ 8 & -1 \end{vmatrix} + (-2)\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 4 \\ 1 & 8 \end{vmatrix}\right) \\
&= (2)(c^3 + c^2) + (4c^3 - 8c^2) + (c)(-4 - 8) \\
&+ (-1)((c^3 + c^2) + (c^3 - c^2) + (c)(-1 - 1)) \\
&+ (4c^3 - 8c^2) + (-2)(c^3 - c^2) + (c)(8 - 4) \\
&+ (-1)((-4 - 8) + (-2)(-1 - 1) + (-1)(8 - 4) \\
&= 6c^3 - 12c^2 - 6c + 12 \\
&= 6(c - 2)(c - 1)(c + 1)
\end{aligned}$$

Therefore, $det(A) = (c - 2)(c - 1)(c + 1) \neq 0$,
$c \neq 2, c \neq 1, c \neq -1$

Actually, this question can be solved by observation. The determinant will be 0 if 2 columns are a multiple of one another. By observation, we can see that if $c = 1$, then the 4th and 1st column will be the same. If $c = 2$, then 2,4 and $c = -1$, then 3,4 will be the same.