# MA1522

## Assignment 1

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# 1 Question 1

$$\left(\begin{array}{ccc|c} 4 & 2 & 4 & 0 \\ 5 & 4 & 0 & 1 \\ 4 & 1 & 2 & 5 \end{array}\right) \xrightarrow{\frac{1}{4}R_1} \left(\begin{array}{ccc|c} 1 & \frac{1}{2} & 1 & 0 \\ 5 & 4 & 0 & 1 \\ 4 & 1 & 2 & 5 \end{array}\right) \xrightarrow[R_3+(-4)R_1]{R_2+(-5)R_1} \left(\begin{array}{ccc|c} 1 & \frac{1}{2} & 1 & 0 \\ 0 & \frac{3}{2} & -5 & 1 \\ 0 & -1 & -2 & 5 \end{array}\right)$$

$$\xrightarrow{\frac{2}{3}R_2} \left(\begin{array}{ccc|c} 1 & \frac{1}{2} & 1 & 0 \\ 0 & 1 & \frac{-10}{3} & \frac{2}{3} \\ 0 & -1 & -2 & 5 \end{array}\right) \xrightarrow{R_3+R_2} \left(\begin{array}{ccc|c} 1 & \frac{1}{2} & 1 & 0 \\ 0 & 1 & \frac{-10}{3} & \frac{2}{3} \\ 0 & 0 & \frac{-16}{3} & \frac{17}{3} \end{array}\right)$$

$$\xrightarrow{\frac{-3}{16}R_3} \left(\begin{array}{ccc|c} 1 & \frac{1}{2} & 1 & 0 \\ 0 & 1 & \frac{-10}{3} & \frac{2}{3} \\ 0 & 0 & 1 & \frac{-17}{16} \end{array}\right) \xrightarrow[R_1+(-1)R_3]{R_2+\frac{10}{3}R_3} \left(\begin{array}{ccc|c} 1 & \frac{1}{2} & 0 & \frac{17}{16} \\ 0 & 1 & 0 & \frac{-23}{8} \\ 0 & 0 & 1 & \frac{-17}{16} \end{array}\right)$$

$$\xrightarrow{R_1+\frac{-1}{2}R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{5}{2} \\ 0 & 1 & 0 & \frac{-23}{8} \\ 0 & 0 & 1 & \frac{-17}{16} \end{array}\right)$$

## 1.1 1i No of pivot columns

There are 3 pivot columns

## 1.2 1ii Arbitrary Params needed

0 arbitrary params needed

## 1.3 1iii How many solutions are there?

1 solution for the system

## 1.4 1iv Solution for system

$x_1 = \frac{5}{2}, x_2 = -\frac{23}{8}, x_3 = -\frac{17}{16}$

## 2 Question 2

$$\left(\begin{array}{ccc|c} a & 1 & 1 & a^3 \\ 1 & a & 1 & 1 \\ 1 & 1 & a & a \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & 1 & a & a \\ 1 & a & 1 & 1 \\ a & 1 & 1 & a^3 \end{array}\right)$$

$$\xrightarrow[R_3+(-a)R_1]{R_2+(-1)R_1} \left(\begin{array}{ccc|c} 1 & 1 & a & a \\ 0 & a-1 & 1-a & 1-a \\ 0 & 1-a & 1-a^2 & a^3-a^2 \end{array}\right)$$

$$\xrightarrow{R_3+(1)R_2} \left(\begin{array}{ccc|c} 1 & 1 & a & a \\ 0 & a-1 & 1-a & 1-a \\ 0 & 0 & 2-a^2-a & a^3-a^2-a+1 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 1 & 1 & a & a \\ 0 & a-1 & 1-a & 1-a \\ 0 & 0 & (a+2)(1-a) & (a+1)(a-1)(a-1) \end{array}\right)$$

### 2.1 2i, system has no solution

The system will have no solution when the last column is the pivot column. This can happen in $R_2$ if $a-1 = 1-a = 0$ and $1-a \neq 0$

$a = 1$ will satisfy the left equation, but it contradicts with the right equation. Thus, $R_2$ is not a possible candidate for no solution

For $R_3$, $(a+2)(1-a) = 0$ and $(a+1)(a-1)(a-1) \neq 0$

Left equation:

$$(a+2)(1-a) = 0$$
$$a = 1 \text{ or } a = -2$$

Right Equation:

$$(a+1)(a-1)(a-1) \neq 0$$
$$a \neq 1 \text{ or } a \neq -1$$

Therefore, there is no solution to the equation if $a = -2$

### 2.2 2ii, system has unique solution

The system has a unique solution when the last column is non pivot and all other columns are pivot columns.

$C_1$ is pivot column, $C_2$ is pivot if $a-1 \neq 0$, $C_3$ is pivot if $(a+2)(1-a) \neq 0$

$$
\begin{aligned}
a - 1 &\neq 0 \\
a &\neq 1 \\
(a + 2)(1 - a) &\neq 0 \\
a \neq -2 \text{ and } a &\neq 1
\end{aligned}
$$

Therefore, the system has a unique solutions when $a \neq 1$ and $a \neq -2$

## 2.3 2iii System has infinitely many solutions

The system will have infinitely many solutions when the last column is non-pivot and some other columns are non-pivot columns.

if $C_2$ is non pivot, then $C_2R_2$ must be 0. $a - 1 = 0, a = 1$

If $C_3$ is non-pivot, then $C_3R_3$ must be zero, but that will cause the last row to be a pivot.

Therefore, the system has infinitely many solutions when $a = 1$