Function and Limits

- $\lim\limits_{x\to\pm\infty} \frac{Ax^\alpha}{Bx^\beta} \begin{cases} 0 & \text{if} \alpha < \beta \\ \frac{A}{B} & \text{if} \alpha = \beta \\ \pm\infty & \text{if} \alpha > \beta \end{cases}$
- $\lim\limits_{x\to c} \frac{sin(g(x))}{g(x)} = 1 (\lim\limits_{x\to c} g(x) = 0)$
- $\lim\limits_{x\to c} \frac{tan(g(x))}{g(x)} = 1$
- $\lim\limits_{x\to 0} \frac{sin(x)}{x} = 1$
- $\lim\limits_{x\to 0} \frac{tan(x)}{x} = 1$

Differentiation

parametric differentiaton: $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx}) = \frac{\frac{d}{dt}(\frac{dy}{dx})}{\frac{dx}{dt}}$

$f(x)$	$f'(x)$
$\tan x$	$\sec^2 x$
$\csc x$	$-\csc x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\csc^2 x$
$a^{f(x)}$	$\ln a \cdot f'(x) a^{f(x)}$
$\log_a f(x)$	$\log_a e \cdot \frac{f'(x)}{f(x)}$
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}, \quad \lvert f(x)\rvert<1$
$\cos^{-1} f(x)$	$-\frac{f'(x)}{\sqrt{1-[f(x)]^2}}, \quad \lvert f(x)\rvert<1$
$\tan^{-1} f(x)$	$\frac{f'(x)}{1+[f(x)]^2}$
$\cot^{-1} f(x)$	$-\frac{f'(x)}{1+[f(x)]^2}$
$\sec^{-1} f(x)$	$\frac{f'(x)}{\lvert f(x)\rvert\sqrt{[f(x)]^2-1}}$
$\csc^{-1} f(x)$	$-\frac{f'(x)}{\lvert f(x)\rvert\sqrt{[f(x)]^2-1}}$

Second Derivative Test: $f'(c) = 0, f''(c) < 0$ then local max, $f''(c) > 0$ local min.

L'Hopital's Rule: Given $\lim\limits_{x\to c} f(x)$ and $g(x) = 0/\pm\infty$

$\lim\limits_{x\to c} \frac{f(x)}{g(x)} = \lim\limits_{x\to c} \frac{f'(x)}{g'(x)}$

- Use for $\frac{0}{0}$ or $\frac{\infty}{\infty}$

Trigo Identities

1. $\sec^2 x - 1 = \tan^2 x$
2. $\csc^2 x - 1 = \cot^2 x$
3. $\sin A \cos A = \frac{1}{2}\sin 2A$
4. $\cos^2 A = \frac{1}{2}(1+\cos 2A)$
5. $\sin^2 A = \frac{1}{2}(1-\cos 2A)$
6. $\sin A \cos B = \frac{1}{2}(\sin(A+B)+\sin(A-B))$
7. $\cos A \sin B = \frac{1}{2}(\sin(A+B)-\sin(A-B))$
8. $\cos A \cos B = \frac{1}{2}(\cos(A+B)+\cos(A-B))$
9. $\sin A \sin B = \frac{1}{2}(\cos(A+B)-\cos(A-B))$

Integration

$f(x)$	$\int f(x)$
$\tan ax$	$\frac{1}{a}\ln\lvert\sec(ax)\rvert$
$\cot ax$	$\frac{1}{a}\ln\lvert\cot(ax)\rvert$
$\sec ax$	$\frac{1}{a}\ln\lvert\sec(ax)+tan(ax)\rvert$
$\csc ax$	$\frac{1}{a}\ln\lvert\csc(ax)+cot(ax)\rvert$
$\frac{1}{a^2+(x+b)^2}$	$\frac{1}{a}\tan^{-1}(\frac{x+b}{a})$
$\frac{1}{\sqrt{a^2-(x+b)^2}}$	$\sin^{-1}(\frac{x+b}{a})$
$\frac{1}{a^2-(x+b)^2}$	$\frac{1}{2a}\ln\lvert\frac{x+b+a}{x+b-a}\rvert$
$\frac{1}{(x+b)^2-a^2}$	$\frac{1}{2a}\ln\lvert\frac{x+b-a}{x+b+a}\rvert$

Substitution $\int f(g(x))\cdot g'(x)dx = \int f(u)du, u = g(x)$
By Parts $\int uv'dx = uv - \int u'vdx$, order: LIATE: Differentiate to integrate

Application of Integration

about x axis

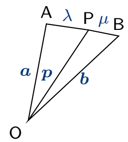
- Vol Disk: $V = \pi\int_a^b f(x)^2 - g(x)^2 dx$
- Vol Shell: $V = 2\pi\int_a^b x\lvert f(x)-g(x)\rvert dx$ (absolute!!)
- Length of curve: $\int_a^b \sqrt{1+f'(x)^2}dx$

Series

TODO!!

Vectors

unit vector: $\hat{p} = \frac{p}{|p|}$, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$



ratio theorem
$\boldsymbol{p} = \frac{\mu\boldsymbol{a}+\lambda\boldsymbol{b}}{\lambda+\mu}$

midpoint theorem
$\boldsymbol{p} = \frac{\boldsymbol{a}+\boldsymbol{b}}{2}$

Dot Product

- $\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3 = |a||b|\cos\theta$
- $a \perp b \Rightarrow a\cdot b = 0$
- $a \parallel b \Rightarrow a\cdot b = |a||b|$

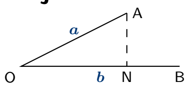
Cross Product

$\boldsymbol{a}\times\boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_i & a_2 & a_3 \\ b_i & b_2 & b_3 \end{vmatrix} = \begin{pmatrix} (a_2b_3 - a_3b_2) \\ -(a_1b_3 - a_3b_1) \\ (a_1b_2 - a_2b_1) \end{pmatrix}$

$|\boldsymbol{a}\times\mathbf{b}| = |\boldsymbol{a}||\mathbf{b}|\sin\theta$
$a \perp b \Rightarrow a\times b = |a||b|$

$a \parallel b \Rightarrow a\times b = 0$
Parallelogram $= |\boldsymbol{a}\times\mathbf{b}|$

Projection



$\triangle ANO = \frac{1}{2}|\overrightarrow{OA}\times\overrightarrow{ON}|$

$\text{comp}_{\boldsymbol{b}}\boldsymbol{a} = |\boldsymbol{b}|\cos\theta = \frac{\boldsymbol{a}\cdot\boldsymbol{b}}{|\boldsymbol{a}|}$
$\text{proj}_{\boldsymbol{b}}\boldsymbol{a} = \text{comp}_{\boldsymbol{b}}\boldsymbol{a}\cdot\frac{a}{|a|} =$
$\overrightarrow{ON} = \frac{\boldsymbol{a}\cdot\boldsymbol{b}}{\boldsymbol{a}\cdot\boldsymbol{a}}\boldsymbol{a} = \frac{\boldsymbol{a}\cdot\boldsymbol{b}}{|\boldsymbol{a}|^2}\boldsymbol{b}$

Lines

$\boldsymbol{r} = \boldsymbol{r}_0 + t\boldsymbol{v} = \langle x, y, z\rangle$
$\langle x_0, y_0, z_0\rangle + t\langle a, b, c\rangle$

$\begin{pmatrix} x_0 + at \\ y_0 + bt \\ z_0 + ct \end{pmatrix}$

Planes

$\boldsymbol{n} = \langle a, b, c\rangle, \boldsymbol{r} = \langle x, y, z\rangle, \boldsymbol{r}_0\langle x_0, y_0, c_0\rangle$
Scalar: $\boldsymbol{n}\cdot\boldsymbol{r} = \boldsymbol{n}\cdot\boldsymbol{r}_0$
Cartesian: $ax + by + cz = d$

Distance from Point to Plane

$\frac{|ax_0+by_0+cz_0-d|}{\sqrt{a^2+b^2+c^2}}$

Partial Derivatives

Chain Rule

For $z(t) = f(x(t), y(t))$,
$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$
For $z(s,t) = f(x(s,t), y(s,t))$,
$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$
$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$
Arc Length of $r(t)$: $\int_a^b |\boldsymbol{r}'(t)|dt$

Implicit Differentiation

$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$ $\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$

Directional Derivative

Gradient vector at $f(x,y): \triangle f = f_x\boldsymbol{i} + f_y\boldsymbol{j}$
$D_u f(x,y) = \langle f_x, f_y\rangle\cdot\langle a, b\rangle = \langle f_x, f_y\rangle\cdot\hat{\boldsymbol{u}} = \triangle f\cdot\hat{\boldsymbol{u}}$
(Unit Vector)
Tangent Plane: $\triangle f\cdot\langle x - x_0, y - y_0, z - z_0\rangle = 0$

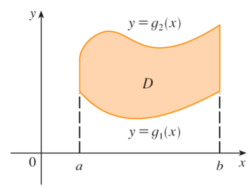
Critical Points

$D = f_{xx}(a,b)f_{yy}(a,b) - (f_{x,y}(a,b))^2$

D	$f_{xx}(a,b)$	**local**
+	+	min
+	-	max
-	any	saddle point
0	any	no conclusion

Double Integrals

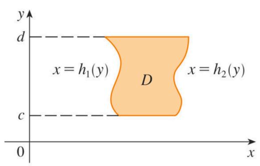
Type I



$\int_a^b\int_{g_1(x)}^{g_2(x)} f(x,y)dydx$

$D = \{(x,y) : a \le x \le b,$
$g_1(x) \le y \le g_2(x)\}$

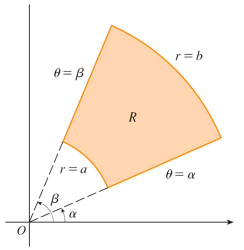
Type II



$\int_c^d\int_{h_1(y)}^{h_2(y)} f(x,y)dxdy$

$D = \{(x,y) : c \le y \le d,$
$h_1(y) \le x \le h_2(y)\}$

Polar Coordinates



$x = r\cos\theta$
$y = r\sin\theta$
$R = \{(r,\theta) : 0 \le a \le r \le b,\ \alpha \le \theta \le \beta\}$

$\int_\alpha^\beta\int_a^b f(r\cos\theta, r\sin\theta)rdrd\theta$

Surface Area

$S = \iint_R \sqrt{f_x^2 + f_y^2 + 1}dA$

ODE

form	change of variable
$\frac{dy}{dx} = f(x)g(y)$	$\int\frac{1}{g(y)}dy = \int f(x)dx + C$
$y' = g(\frac{y}{x})$	Set $v = \frac{y}{x}$ $\Rightarrow y' = v + xv'$
$y' = f(ax+by+c)$ $\Rightarrow y' = \frac{ax+by+c}{\alpha x+\beta y+\gamma}$	Set $v = ax + by$
$y' + P(x)y = Q(x)$	$R = e^{\int P(x)dx}$ $\Rightarrow y\cdot R = \int Q\cdot Rdx$
$y' + P(x)y = Q(x)y^n$	$z = y^{1-n}$ $\Rightarrow$ sub in Z solve linear

Population Models

$N_\infty = \frac{B}{s}$, $\hat{N} =$ Population Now

Malthus
$N(t) = \hat{N}e^{kt}$
$k = B - D$

Logistic
$\frac{1}{N} =$
$\frac{1}{N_\infty} + (\frac{1}{\hat{N}} - \frac{1}{N_\infty})e^{-Bt}$
$N = \frac{N_\infty}{1+(\frac{N_\infty}{\hat{N}}-1)e^{-Bt}}$

Uranium Decay into Thorium

$U(t) = U_0e^{-k_ut}$, $k = \frac{\ln 2}{\text{halflife}}$, $\frac{dU}{dt} = -k_uU$
Thorium:
$T(t) = \frac{K_uU_0}{K_t - K_u}(e^{-k_ut} - e^{-k_tt})$, $\frac{dT}{dt} = k_uU - k_TT$