1. Reference

- Sample Space (S): Set of all possible outcomes of an experiment
  - Can vary depending on the problem of interest
- Sample Point: Outcome of sample Space (Element)
- Event: Subset of sample space (Set)

1.1. Set Operations

- $A \cup B = \{x : x \in A \text{ or } x \in B\}$
- $A \cap B = \{x : x \in A \text{ and } x \in B\}$
- $A'_n = \{x : x \in S \text{ and } x \notin A\}$
- $\bigcup_{i=1} A_i = A_1 \cup A_2 \cup ... \cup A_n = \{x : x \in A_1 \text{ or } \ ... \text{ or } x \in A_n\}$
- Mutually exclusive / disjoint - $A \cap B = \emptyset$
- Contained - All in A are also elements in B, A is contained in B, $A \subset B$ or $B \supset A$
- Equivalent - $A \subset B$ and $B \subset A$, then $A = B$

1.2. Set Operations

- $A \cap A' = \emptyset$
- $A \cap \emptyset = \emptyset$
- $A \cup A' = S$
- $(A')' = A$
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- $A \cup B = A \cup (B \cap A')$
- $A = (A \cap B) \cup (A \cap B')$
- $(A_1 \cup A_2 \cup ... \cup A_n)' = A'_1 \cap A'_2 \cap ... \cap A'_n$
- $(A_1 \cap A_2 \cap ... \cap A_n)' = A'_1 \cup A'_2 \cup ... \cup A'_n$

1.3. Counting Methods

- Multiplication Principle - r different experiments to be performed sequentially. Then there are $n_1 n_2 ... n_r$ possible outcomes for r experiments
- Addition Principle - experiment can be performed by k different procedures. Suppose ways under different procedures **do not overlap**. Then total is $n_1 + ... + n_k$
- **Permutation** is selection of r objects out of n. Order is taken into consideration.

$$P_r^n = \frac{n!}{(n-r)!} = n(n-1)(n-2)...(n-(r-1))$$

  (When $r = n$, $P_n^n = n!$)
- **Combination** is selection of r objects out of n, without regard for order.

$$\binom{n}{r} = \frac{n!}{r!(n-r!)} = \binom{n}{n-r}$$

  - Intuition: In terms of permutation, no of ways to choose and arrange r objects out fo n is P_r^n
  - This can be also done by the following:
    - Select r objects out of n without regard to order: $\binom{n}{r}$ ways
    - For each combination, permute its r objects: P_r^r ways
    - For each combination, permute its r objects: P_r^r ways
    - $\binom{n}{r} \times P_r^r = P_r^n$

1.4. Probability

- Probability is chance or how likely a certain event may occur. Let A be an event in an experiment. $P(A)$ is to quantify how likely A may occur.

1.4.1. Axioms

Probability, $P(\cdot)$ is a function on the collection of events in the sample space satisfying:

- For any event A, $0 \leq P(A) \leq 1$
- For the sample space $P(S) = 1$
- For any 2 mutually exclusive event A and B, that is $A \cap B = \emptyset$, $P(A \cup B) = P(A) + P(B)$
- $P(\emptyset) = 0$
- $P(A') = 1 - P(A)$
- $P(A) = P(A \cap B) + P(A \cap B')$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $A \subset B$, then $P(A) \leq P(B)$